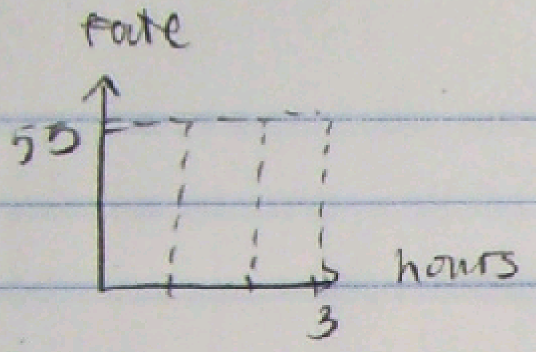
A car drives at a constant rate of 55 mph for 3 hours.

a) Sketch a graph of this situation. Make sure to label your axes.

b) How far did the car travel?

c) How can we see this distance on your graph?

d) How can we see the velocity and accelleration?

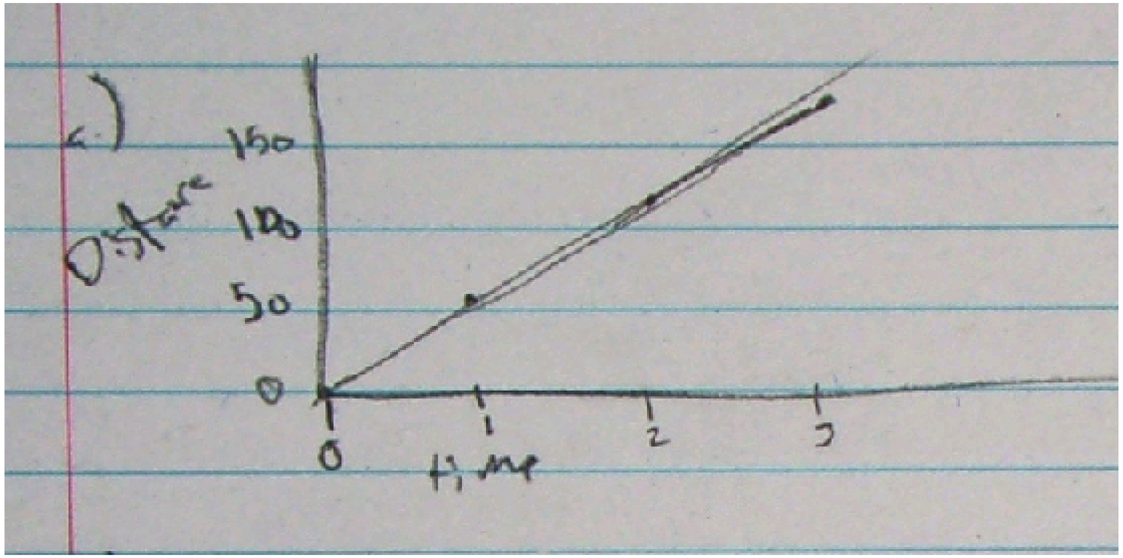


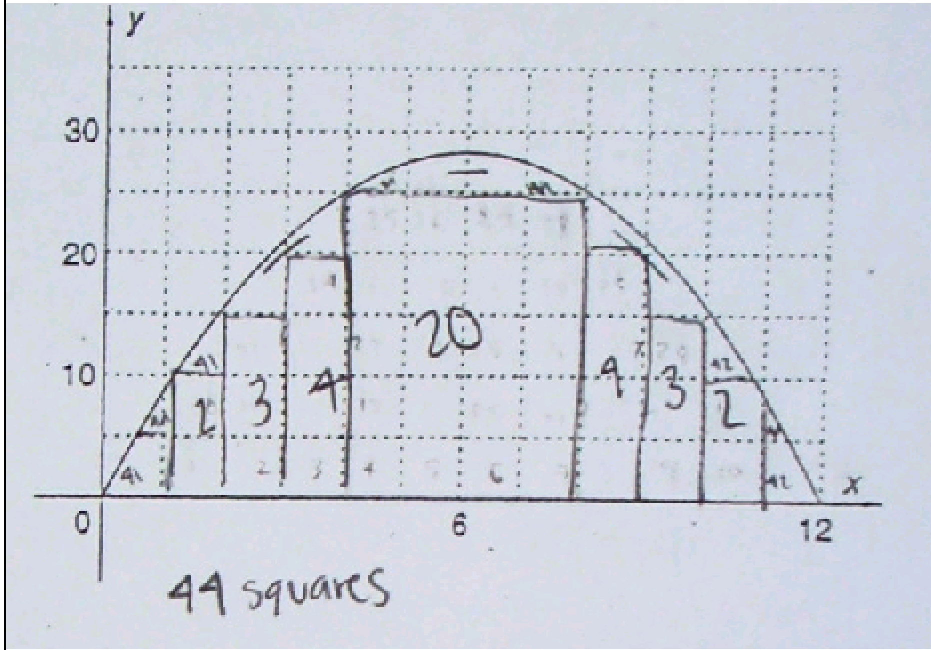
b) The car traveled 165 m

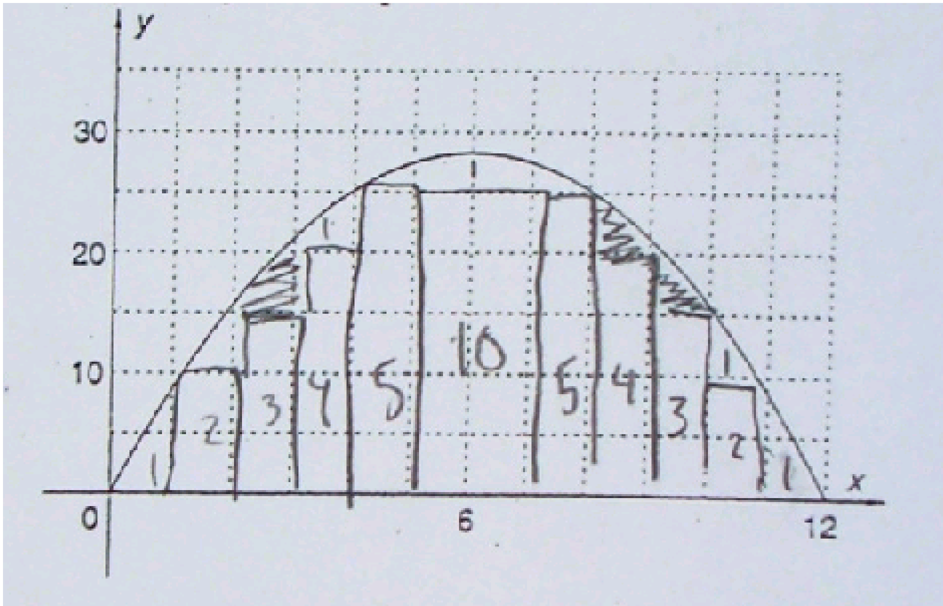
c) Distance is the area of the rectangle on the graph

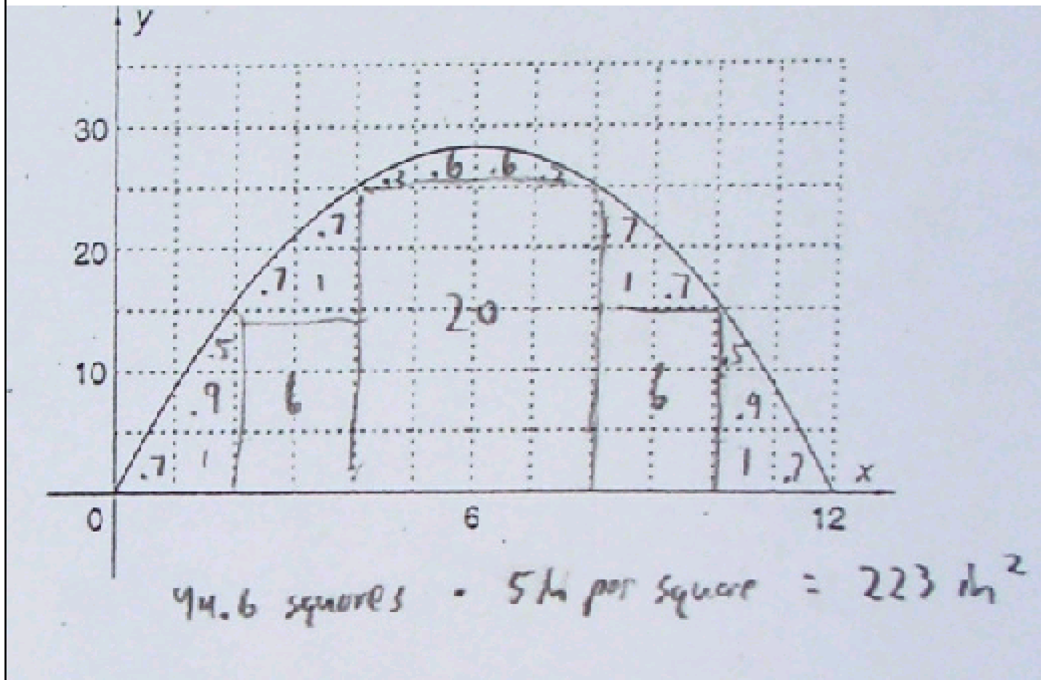
d) the velocity is the flat line at 55mph

acceleration is the graph of 1st deriv ✓




y
30
20
10
0
6
12
x
44 squares

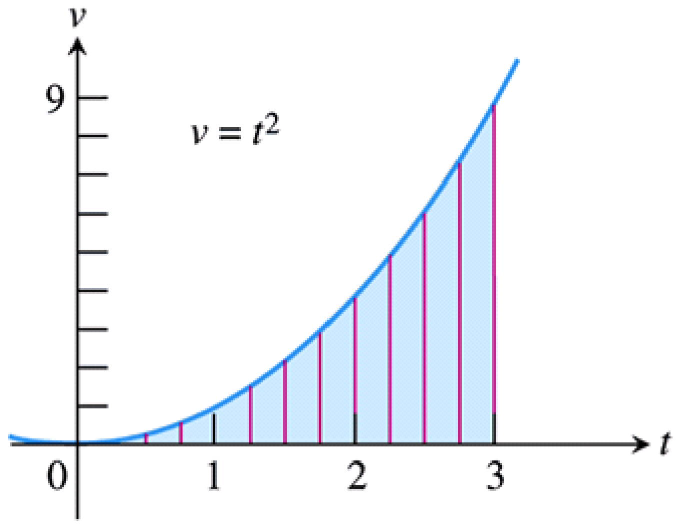

y
30
20
10
0
6
12
x


y
30
20
10
0
6
12
x
44.6 squares · 5 in per square = 223 in²

Example **Finding Distance Traveled when Velocity Varies**

A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2$ for time $t \geq 0$. Where is the particle at $t = 3$?

Graph v and partition the time interval into subintervals of length Δt. If you use $\Delta t = 1/4$, you will have 12 subintervals. The area of each rectangle approximates the distance traveled over the subinterval. Adding all of the areas (distances) gives an approximation to the total area under the curve (total distance traveled) from $t = 0$ to $t = 3$.



Example Finding Distance Traveled when Velocity Varies

Table 5.1

Subinterval	$\left[0, \frac{1}{4}\right]$	$\left[\frac{1}{4}, \frac{1}{2}\right]$	$\left[\frac{1}{2}, \frac{3}{4}\right]$	$\left[\frac{3}{4}, 1\right]$
Midpoint m_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
Height $= (m_i)^2$	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{25}{64}$	$\frac{49}{64}$
Area $= (1/4)(m_i)^2$	$\frac{1}{256}$	$\frac{9}{256}$	$\frac{25}{256}$	$\frac{49}{256}$

Continuing in this manner, derive the area $(1/4)(m_i)^2$ for each subinterval and add them:

$$\frac{1}{256}+\frac{9}{256}+\frac{25}{256}+\frac{49}{256}+\frac{81}{256}+\frac{121}{256}+\frac{169}{256}+\frac{225}{256}+\frac{289}{256}+\frac{361}{256}+\frac{441}{256}+\frac{529}{256}=\frac{2300}{256}$$

$$\approx 8.98$$

5.1
Rectangular Approximation Method (RAM)

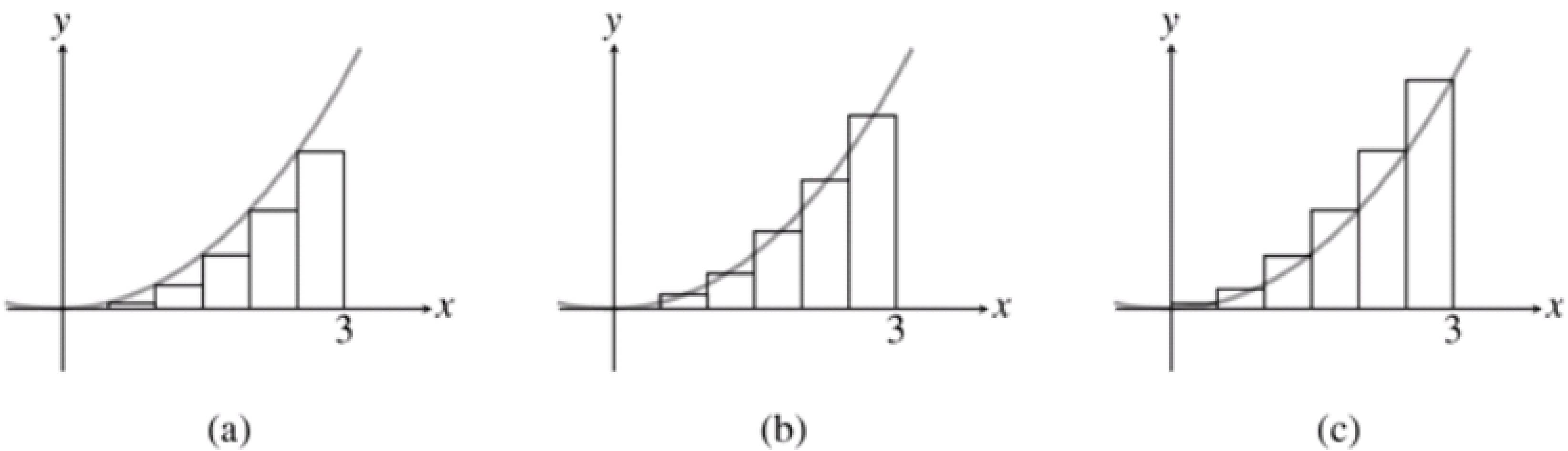


Figure 5.7 The area under the graph of $y = x^2$ from $x = 0$ to $x = 3$ can be approximated using rectangles whose left-hand endpoints are on the graph (LRAM, left diagram), rectangles whose midpoints are on the graph (MRAM, middle diagram), or rectangles whose right-hand endpoints are on the graph (RRAM, right diagram).

No matter which RAM approximation we compute, we are adding products of the form $f(x_i) \cdot \Delta x$, or, in this case, $(x_i)^2 \cdot (3/6)$.

LRAM:

$$(0)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + (1)^2\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right) + (2)^2\left(\frac{1}{2}\right) + \left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right) = 6.875$$

MRAM:

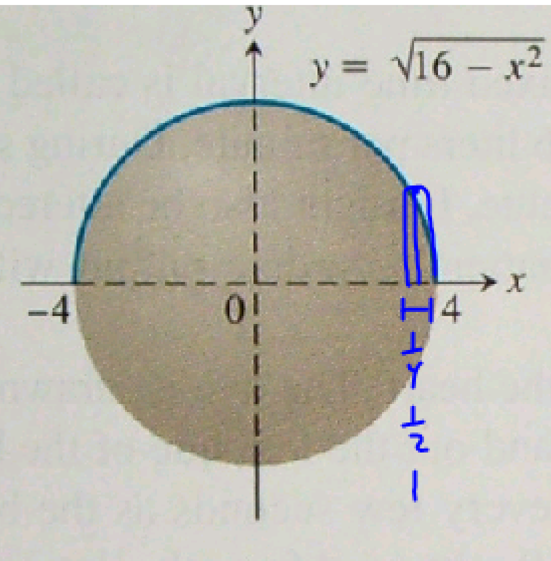
$$\left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{5}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{7}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{9}{4}\right)^2\left(\frac{1}{2}\right) + \left(\frac{11}{4}\right)^2\left(\frac{1}{2}\right) = 8.9375$$

RRAM:

$$\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + (1)^2\left(\frac{1}{2}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{2}\right) + (2)^2\left(\frac{1}{2}\right) + \left(\frac{5}{2}\right)^2\left(\frac{1}{2}\right) + (3)^2\left(\frac{1}{2}\right) = 11.375$$

As we can see from Figure 5.7, LRAM is smaller than the true area and RRAM is larger. MRAM appears to be the closest of the three approximations. However, observe what happens as the number n of subintervals increases:

n	LRAM_n	MRAM_n	RRAM_n
6	6.875	8.9375	11.375
12	7.90625	8.984375	10.15625
24	8.4453125	8.99609375	9.5703125
48	8.720703125	8.999023438	9.283203125
100	8.86545	8.999775	9.13545
1000	8.9865045	8.99999775	9.0135045

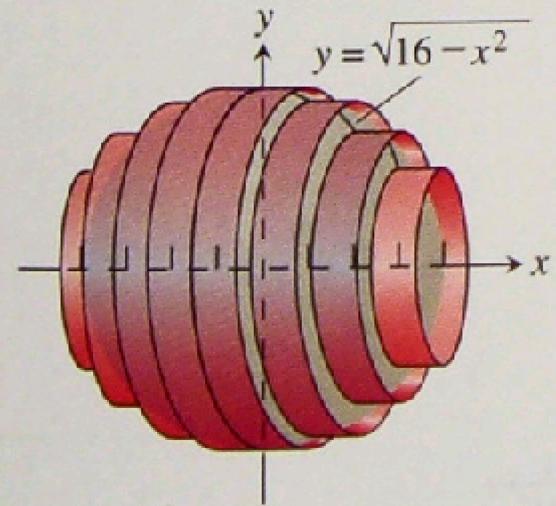


(a)

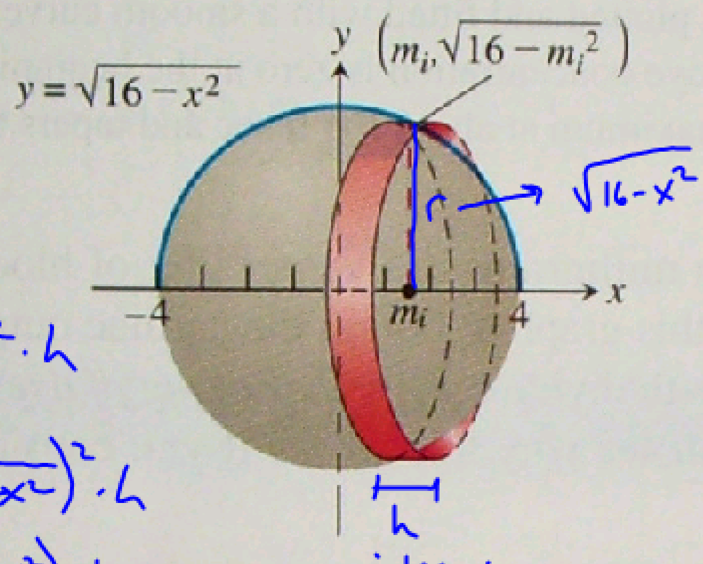
$V = \pi r^2 \cdot h$

$V = \pi \left(\sqrt{16 - x^2}\right)^2 \cdot h$

$V = \pi (16 - x^2) \cdot h$



(b)



(c)

Figure 5.9 (a) The semicircle $y = \sqrt{16 - x^2}$ revolved about the x-axis to generate a sphere. (b) Slices of the solid sphere approximated with cylinders (drawn for $n = 8$). (c) The typical approximating cylinder has radius $f(m_i) = \sqrt{16 - m_i^2}$. (Example 3)

HW: Sect. 5.1 #1,3,5, 14(only $n=10$),16, 18-20,23,27